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# TWO-MESON AND MULTI-PION FINAL STATES FROM 600 GeV PION INTERACTIONS

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## ABSTRACT

This report describes the transitions $\pi^- \to meson_1 + meson_2$ and also $\pi^- \to multi - \pi$ for high energy pions interacting with target nuclei (Z,A). The physics interests are: A) Nuclear inelastic coherent diffraction cross sections for pions, for studies of size fluctuations in the pion wave function. B) Radiative widths of excited meson states, for tests of vector dominance and quark models. C) Experimental determination of the $\pi^- + \rho \to \pi^- + \gamma$ total reaction rate for photon production above 0.7 GeV, needed for background studies of quark-gluon plasma formation experiments. D) Investigation of the $\gamma \to 3\pi$ vertex in pion pair production by a pion, for a significantly improved test of the hypothesis of chiral anomalies. The physics interest and associated bibliography are summarized here; with reference to the 200-600 GeV beams available at CERN and FNAL. Complementary GEANT simulations and trigger studies are needed.

# INTRODUCTION

There are a number of different physics objectives (A-D, below) for studies in E781 [1] and elsewhere of the reactions 1-8 listed below, reactions involving mainly pion induced two-meson final states. The discussion here describes the different objectives separately, and the relevant data needed for each objective. The objectives are: A) nuclear inelastic coherent diffraction cross section channels for incident pions or hadrons, for studies of color fluctuations. B) Radiative widths of excited Meson States Via Primakoff. C) determination of the $\pi^- + \rho \to \pi^- + \gamma$ total reaction rate for photon production above 0.7 GeV, via Primakoff. D) Investigation of the Chiral Anomaly $\gamma \to 3\pi$ in pion pair production by a pion, via Primakoff. The Primakoff studies B,C,D are rather straightforward, the diffractive studies need to be studied as background to the Primakoff reactions; but here are discussed for their own merits. Detailed discussions of items A-D are given below.

Examples of the reactions considered are:

1) $\pi^-$ + virtual photon $\to \pi^- + \rho$ (Primakoff)

2) $\pi^-$ + pomeron $\to \pi^- + \rho$ (Diffractive)

3) $\pi^-$ + virtual photon $\to K^- + K^*$ (Primakoff)

4) $\pi^-$ + pomeron $\to K^- + K^*$ (Diffractive)

5) $\pi^-$ + virtual photon $\to \pi^- + \omega$ (Primakoff)

6) $\pi^-$ + pomeron $\to \pi^- + \omega$ (diffractive, not allowed by G-parity)

7) $\pi^-$ + pomeron $\to$ L pions + M kaons

8) hadron + pomeron $\to$ hadron' meson (for example, proton $\to$ Lambda K$^+$, or $\pi^- \to \bar{p}p\pi^-$)

Some quantum numbers relevant to the above reactions are:

pion: C=+1, G-parity=-1, Isospin=1

rho: C=-1, G=1, I=1

omega: C=-1, G=-1, I=0

pomeron: C=+1, G=1

$a_1(1260)$: C=+1, G=-1, I=1, pi-rho decay dominant

$b_1(1235)$: C=-1, G=+1, I=1, pi-omega decay dominant

gamma: G=-1,I=0 or G=+1,I=1

We distinguish between Primakoff production of a final state $\pi\rho$ configuration, via the reaction $\pi$ + virtual photon $\rightarrow \pi+\rho$; and a diffractive process, which involves $\pi$ + pomeron $\rightarrow \pi+\rho$. Both Primakoff production and diffractive production give events at small t, the four-momentum transfer to the target nucleus. A simple cut on t is not the best way to make the separation. Rather, the t-distribution d$\sigma$/dt must be fit in terms of: $d\sigma/dt = F_A^2(t)(ZC_1/\sqrt{t}+C_2exp(at))^2$; where $F_A$(t) is the nuclear form factor. The Coulomb Primakoff events follow the $C_1$ term, and the diffractive pomeron events follow the $C_2$ term. For the pomeron events only, $d\sigma/dt = F_A^2(t)C_2^2exp(12t) \sim C_2^2$ for t$R_A^2/3 << 1$. Zielinski et al. [2] did not include an interference term for three pion production, since Coulomb-produced final states have Gottfried- Jackson helicity M=$\pm$1, while strong production occurs dominantly with M=0. We consider here only the coherent Primakoff and diffractive cross sections, in which the value t is restricted to the coherent region. Zielinski et al. [2] define the coherent region for three pion production as $t < t*$ where $t* = 0.4A^{-2/3}GeV^2$ for incident 200 GeV pions. The value t* corresponds to the expected first minimum of the t-distribution for a target of nucleon number A. Similar t* definitions will be used for other final diffractive states. This work shows how one can attain the required t-resolution needed to guarantee coherent diffraction. For a target nucleus with A=208, the maximum nuclear recoil energy is $E* = t*/(2 \times 208 \times 0.93) = 30$ KeV, below the excitation energy of low lying nuclear levels.

For the diffractive data, following separation from the Coulomb cross section, one

is left with the difficult task of estimating the acceptance and reconstruction efficiency. The efficiency is sensitive to the polar angle distribution of the decay pions, which is different for different possible spin-parity states of the decaying system. A spin-parity analyses is done [2, 3], which is also sensitive to the number of terms in the model. Usually spins greater than J=3 are ignored; otherwise there are no convergent fits. This analysis was carried out for the three-pion case, but would be more difficult for more complex final states. In addition, the three-pion analysis was restricted to $M_{3\pi}$ values between 0.8 and 1.5 GeV. For larger masses, the f$\pi$ decay mode becomes important, and the partial wave structure is more complex. The efficiencies were determined in this way to $\pm 10\%$ for the three-pion case. Similar analyses are required for each diffractive channel and mass range measured.

We consider diffractive events proceeding through pomeron exchange. For the $\pi^-$ pomeron $\rightarrow \pi^- \rho$ transition, G-parity is negative for initial and final states. For the $\pi^-$ pomeron $\rightarrow \pi^- \omega$ transition, G-parity is negative for the initial state, and positive for the final state. Therefore, the soft cross section is dominated by photon exchange for the $\pi\omega$ final state. By studying both $\pi\rho$ and $\pi\omega$ final states, we can better learn how to separate Primakoff from diffractive events. The $\omega$ is observed via it decay to $\pi^+\pi^-\pi^0$. The invariant mass spectra of the $\pi\omega$ and $\pi\rho$ systems produced are important, and will also be studied.

**A) Inelastic Coherent Diffraction Cross Sections.**

Soft coherent diffractive dissociation of an incident pion by a nuclear target can provide important experimental tests of the idea of size fluctuations in the projectile wave function [4, 5, 6, 7, 8, 9]. The target remains in its ground state, as the incident pion diffractively dissociates. The incident pion can be considered as a superposition of different configurations, having different sizes. Large inelastic diffractive cross sections arise only if there are significant differences in the absorption cross sections of the different configurations, as described in references [4, 8] and references therein. For example, the pion wave function can be expanded into states of $q\bar{q}$, $q\bar{q}g$, $qq\bar{q}\bar{q}$, $qq\bar{q}\bar{q}g$, $q\bar{q}gg$, etc. Some of these Configurations such as $q\bar{q}$ have Small Size, some have Normal Size, some have Large Size. These are labelled as SSC, NSC, LSC.

The time scale for fluctuations of the incident pion of mass m into an excited state of mass M is given by the uncertainty principle, as $\tau \sim \hbar/(E(m) - E(M))$. For large $p_{lab}$, the energy denominator $\approx (m^2 - M^2)/2p_{lab}$ is small, and the fluctuation time is long. The excited state M can move a considerable distance before decaying, the coherrence length $l_c = 2p_{lab}/(M^2 - m^2)$, greater than the diameter of a target nucleus [4]. The interactions occur between the excited configuration and target material over the coherrence length, so that the amplitudes from the entire target for the diffractive dissociation add coherently and constructively. The incident pion, entering the nucleus in a specific initial configuration, can be treated as frozen in that configuration as it passes through the entire nucleus.

FMS [4, 8] described a two component model of the pion projectile, to illustrate the basic idea of diffraction as formulated by Feinberg and Pomeranchuk [10], and by Good and Walker [11]. The two component wave function is taken here as:
$\mid \pi >= a \mid SSC > +b \mid NSC >$. If the two components are absorbed with equal strength $\epsilon$, the final state is just $\epsilon \mid \pi >$, and no inelastic states are produced. Otherwise, the final state does not coincide with $\mid \pi >$ and inelastic diffraction takes place [4]. The interaction between a SSC pion and the target is weak, because color fields in the closely packed SSC cancel each other. The term *color fluctuations* is used to describe how the pion fluctuates between its various configurations, and how color dynamics affects the interaction strengths of the different configurations [4]. For simplicity, we do not discuss here a number [4] of possible dynamical mechanisms for the different strengths of different configurations other than size fluctuations. In this framework, if the inelastic diffraction cross sections are large, then the pion wave function must have significant size fluctuations, which is in line with intuition based on quark models of a hadron. Understanding such fluctuation effects is simplest at high energies, for which the coherence length can be significantly larger than the nuclear diameter. But energy dependent experiments could still be of great value in showing the onset of the coherrence effects.

High energy diffractive processes have been described [4] in terms of a probability $P(\sigma)$ that a configuration interacts with a cross section $\sigma$. $P(\sigma)$ estimated from data

is broad; in line with the view that different size configurations interact with widely varying strengths or cross sections. One can describe $P(\sigma)$ in terms of its moments: $\langle\sigma^n\rangle = \int \sigma^n P(\sigma) d\sigma$. The zeroth moment is unity, by conservation of probability, and the first corresponds to the total hadron-nucleon cross section $\sigma_{tot}$ (hN). The second moment has been determined from available diffractive dissociation data. Different determinations [4, 8] give consistent values for $\langle\sigma^2\rangle$, the variance of the distribution: $\omega_\sigma \equiv (\langle\sigma^2\rangle - \langle\sigma\rangle^2)/\langle\sigma^2\rangle$ , with $\omega_\sigma(p) \sim 0.25$ and $\omega_\sigma(\pi) \sim 0.4$, for incident momenta near 200 GeV/c. The part of the pion total cross section associated with SSC is roughly 2-5%, the integral of P($\sigma$) for $\sigma$ values less than 5 mb [4]. It has been suggested [12] to look for the SSC via the hard dissociation of a $q\bar{q}$ pion to two mini-jets. But it is useful to consider also whether one can identify SSC contributions in other reaction channels, including those considered here.

Nuclear hadron inelastic coherent diffractive cross sections provide experimental tests of size or color fluctuations. The total diffractive cross section for an incident pion is given as [4]:

$$\sigma_{diff}(A) = \int d^2B \times \left[\int d\sigma P(\sigma) \sum_n [<\pi \mid F(\sigma,B) \mid n>^2] - [\int d\sigma P(\sigma) <\pi \mid F(\sigma,B) \mid \pi>]^2\right]. \quad (1)$$

Here $F(\sigma,B) = 1 - e^{-\frac{1}{2}\sigma T(B)}$, $T(B) = \int_{-\infty}^{\infty} \rho_A(B,Z)dz$, with $\rho_A(B,Z)$ the nuclear density. The direction of the beam is $\hat{Z}$ and the distance between the projectile and the nuclear center is $\vec{R} = \vec{B} + Z\hat{Z}$. Related equations were given previously within the two-gluon exchange model by Kopeliovich et al. [13]; and within the framework of color transparency by Bertsch et al. [14].

For the extreme black disk limit, the function $F(\sigma,B)$ in Eq. 1 is unity for positions inside the nucleus and zero otherwise, so that $\sigma_{diff}$ vanishes [4]. One can also show that color fluctuations lead to non-vanishing diffractive cross sections by considering the consequences of ignoring the fluctuations of the cross sections in Eq. 1. In that case, $P(\sigma)$ is a delta function, $P(\sigma) = \delta(\sigma - \sigma_{tot}(\pi N))$, and this leads [4] again to $\sigma_{diff}(A) = 0$.

Such vanishing cross sections contrast strongly with the color fluctuation calcu-

lation of FMS [4] using realistic $P(\sigma)$. The FMS calculation for the total diffractive cross section leads to $\sigma^{pA}_{diff}(A) \propto A^{0.80}$ for $A \sim 16$ and $A^{0.4}$ for $A \sim 200$. For the pion projectile, FMS find $\sigma^{\pi-A}_{diff}(A) \propto A^{1.05}$ for $A \sim 16$ and $A^{0.65}$ for $A \sim 200$. The FMS calculation agrees well with the A-dependence of semi-inclusive data of [15] on $n+A \rightarrow p\pi^- + A$ for $p_n \sim 300 GeV$ and of [2] on $\pi^+ + A \rightarrow \pi^+ + \pi^+ + \pi^- + A$ for $p_{\pi^+}$ = 200 GeV. The major part of the inelastic diffractive cross section $\sigma$(diff) and its A-dependence come from configurations centered around those of normal size pions and neutrons [4].

The FMS calculation predicts [4] very large diffractive cross sections; 40-50 mb for $\pi$-Nucleus and n-Nucleus interactions at 200 GeV at A $\sim$ 100 and p$_\pi$ $\sim$ 200 GeV/c. The specific observed pion and neutron channels described above correspond only to roughly 20% and 4% respectively, of this prediction [4]. A proper demonstration of the success of the color fluctuation predictions requires experimentally measuring the cross section and A-dependence of a larger percentage of the total diffractive cross section. Such future experiments should measure soft coherent diffractive dissociation of a pion to L-pion final states with L=3,5,7,9 charged or charged plus neutral pions; mixed pion/kaon final states with L pions and M kaons, two-meson final states such as $\pi^-\rho$ and $K^{0*}$K$^-$, as well as baryonic states such as $p\bar{p}L\pi$. Other channels, discussed by Good and Walker [11], are $\pi \rightarrow K\bar{K}^0$, $\pi \rightarrow K\bar{K}^0\pi^0$, $\pi \rightarrow \bar{p}n$, $\pi \rightarrow \bar{\Lambda}\Sigma$. It is important to get improved data compared to Zielinski et al. [2] for the three-pion case, and new data for the many other channels. Soft diffractive dissociation cross section data for other hadrons are also of great interest; as for $\Sigma^- \rightarrow \Lambda\pi^-$ and other $\Sigma$ channels that can be studied in E781. Such new experiments will also thereby determine how the total diffractive cross section is distributed between the different possible channels [4]. Theoretical prediction are needed for such channel-distributions; they are not yet available. Such data and calculations may further clarify our understanding of color fluctuations.

At a given pion beam energy, a simple guess is that the same A-dependence is expected for all the diffractive channels. It would be surprising and interesting if a particular semiexclusive reaction would have a very different A-dependence from that

predicted for the total $\sigma$(diff). That would open the posssibility, not predicted, that such a particular channel could be more sensitive to SSC or LSC. Different semiexclusive reactions can be studied in E781. A variety of different targets are needed to be sensitive to the interesting change predicted in the A-dependence between light and heavy targets. The A-dependence for reactions discussed above can be further subclassified according to the final state invariant and transverse masses. The reactions should also be studies on the proton target. Aside from the extension of the A-dependence to A=1, such data are the natural input for competing theoretical approaches for hadron-Nucleus interactions, based on the usual quasi-free approximations.

We discuss now in more detail detecting $\pi^-\rho$ and $K^{0*}$K$^-$ two-meson final states, where the two outgoing mesons have roughly equal and opposite p$_T$. The $\rho$ and K* are easy to detect, with $\rho \rightarrow \pi^+\pi^-$ and $K^{0*} \rightarrow \pi^- K^+$. We have relative transverse momentum $\Delta(p_T)$ for the two outgoing mesons, and total transverse momentum transfer $\Sigma p_T$ to the target nucleus. Extremely small momentum transfer $t << 3/R_A^2$ are needed for coherence to be valid [4]. For large $\Sigma$p$_T$, the final state nucleus can be in an excited state, which would destroy coherence. The relative transverse mass m* is given by m$^{*2} \approx 4\Delta^2(p_T)$ for a two-particle final state. Consider experiments such as E781 for which the incident energy satisfies $E >> 2p_T^2 R_A$ or equivalently $E >> m*^2\ R_A/2$, where the transverse mass is m* is 0.6-1.0 GeV and R$_A$ is the nuclear radius. Using 1 fm=5 (GeV/c)$^{-1}$, with R$_A$=25 (GeV/c)$^{-1}$ for a 5 fm target radius, and m*=1GeV, this condition corresponds to $E >> 12.5$ GeV. The 1 GeV transverse mass condition corresponds to an upper limit on $\Delta(p_T)$ of 1 GeV/c, a soft process.

Each configuration of the pion wave function can contribute to a two-meson final state, in which each meson has p$_T$ in opposite directions. The $q\bar{q}$ component can emit the $q$ and $\bar{q}$ with opposite P$_T$, and then pick up a slow $q\bar{q}$ from the vacuum. For the $q\bar{q}$-glue component, there are other possibilities. The $q$ and $\bar{q}$ can be emitted with opposite P$_T$, together with a soft gluon. This gluon can itself dissociate to $q\bar{q}$, which can join the original $q\bar{q}$ to form the final meson$_1$ and meson$_2$.

Different channels can certainly have different relative contributions from different pion configurations. Consider the $d\bar{u}$ component of the pion as an example. Studies of the structure function of the kaon show that the s-quark carries the larger fraction of the kaon momentum. The $d\bar{u}$ of the $\pi^-$ separate, and say combine with an $s\bar{s}$ from the vacuum, to form $d\bar{s}$ $K^{0*}$ and $s\bar{u}$ (K$^-$). The overlap with the final kaon wave functions may be very small; since the produced $s\bar{s}$ pair is slow, but the strange quarks in the final state kaons should be leading. This is not the case for the $q\bar{q}g$ component. Here, the momentum fraction x(g) of the gluon may be large, the gluon may dissociate to $s\bar{s}$, and the resulting s quarks may be leading. For $\pi^- \rightarrow \pi^-\rho$, the $d\bar{u}$ of the $\pi^-$ separate, and say combine with a $u\bar{u}$ from the vacuum, to form $d\bar{u}$ $(\pi^-)$ and $\rho^0$ $(u\bar{u})$. In this case, there may be better overlap with the final state $\pi$ and $\rho$ wavefunctions. The $u$ and $\bar{u}$ do not have to be leading in the final mesons. This example illustrates that different configurations of the pion, say $d\bar{u}$, can contribute differently to different two-meson channels. It is possible therefore that some particular channel (or mass region) may have a different A-dependence than others, or may be more or less sensitive to SSC.

The total soft diffractive cross section probability for a pion can be approximated by [4]:

$$\sigma_{diff}^{appr}(A) = (\omega_\sigma \langle \sigma^2 \rangle / 4) \int d^2 B T^2(B) exp[-\sigma_{tot}(hN) T(B)]. \tag{2}$$

This expression is found [4] starting with Eq. 1, and expanding P($\sigma$) in a Taylor series around $\bar{\sigma} = \sigma_{tot}(hN)$. This formula in the framework of the color fluctuations calculation, gives a clear relationship between the measured total diffractive cross section for target A, and the total hadron-Nucleon cross section. A confirmation of this relationship would provide another test for the color fluctuation framework. The color fluctuation framework also gives predictions [4] for the zero-degree total diffractive differential cross sections, which can provide yet further checks.

The 1-3 GeV higher transverse mass range for two-meson diffractive transitions involves a mixture of soft and hard processes. Theoretical calculations are difficult for this mixed region. It is difficult to get high quality data for transverse mass values

significantly higher than 1 GeV, considering that the cross sections and statistics will be too small. Some low statistics data for this mass range may nonetheless motivate theoretical efforts. One may expect [16] also that two-meson data at higher transverse mass would be more sensitive to the SSC, as was demonstrated recently for two-jet [12] diffraction. A natural guess for fixed invariant mass M is that the transparency and A-dependence will increase with increasing transverse mass [16, 4]. One may also expect [14] that diffractive production of charm may be sensitive to SSC.

**B) Radiative Widths of Mesons.**

High energy pion experiments at FNAL and CERN can obtain new high statistics data for radiative transitions leading from the pion to the $\rho$, to the $a_1(1260)$, and to the $a_2(1320)$. These radiative transition widths are predicted by vector dominance and quark models. They were studied in the past by different groups, but independent data would still be of value. For $\rho \to \pi\gamma$, the widths obtained [17, 18, 19, 20] range from 60. to 81. KeV. For $a_1(1260) \to \pi\gamma$, the width given [21] is $\Gamma = 640. \pm 246.$ KeV; for $a_2(1320) \to \pi\gamma$, the width given [22] is $\Gamma = 295 \pm 60$ KeV; and for $b_1(1235)$, the width given [3] is $\Gamma = 230 \pm 60$ KeV. Radiative transitions of $\Sigma \to \Sigma*$ can also be studied [23, 24]. The $a_1(1260)$ radiative width is related to the pion polarizability [23, 25, 26].

Studies are also possible for Primakoff production of exotic mesons. Consider the search for C(1480) $1^{--}$ via Primakoff production; described by L. Landsberg [27]. It involves Primakoff production of C(1480) with a $\pi^-$ beam at 600 GeV, and observation of the C decay by:

a) C $\to \phi\pi^-$ (with $\phi \to K^-K^+$)

b) C $\to \omega\pi^-$ (with $\omega \to \pi^+\pi^-\pi^0$; and $\pi^0 \to \gamma\gamma$).

c) C $\to \omega\pi^-$ (with $\omega \to \pi^0\gamma$ ; and $\pi^0 \to \gamma\gamma$).

Consider also the search for $1^{-+}$ Hybrid Mesons, as described by M. Zielinsky et al. [28]. One can look for Primakoff production with a 600 GeV $\pi^-$ beam of the Hybrid meson HY, via observation of a number of decay channels:

a) HY $\to \pi^- f1(1285)$ (f1(1285) $\to \pi^+\pi^-\eta; \eta \to \gamma\gamma$)

b) HY $\to \pi^-\eta(\eta \to \gamma\gamma)$

c) HY $\rightarrow \rho^0\pi^-(\rho^0 \rightarrow \pi^+\pi^-)$

d) HY $\rightarrow \eta'\pi^-(\eta' \rightarrow \pi^+\pi^-\eta; \eta \rightarrow \gamma\gamma)$.

**C) Experimental determination of the $\pi + \rho \rightarrow \pi + \gamma$ reaction rate.**

The gamma production reaction can be studied (via the inverse reaction, with detailed balance) via the Primakoff reaction $\pi^- + \gamma \rightarrow \pi^- + \rho^0$. This production rate enters the consideration of the expected gamma-ray background from the hot hadronic gas phase in heavy ion collisions, and is important for quark gluon plasma experimental searches via gamma ray production. Of interest also is the invariant mass of the produced $\pi\rho$ system. The invariant mass reveals information regarding the reaction mechanism. For the case of $\pi\rho$ detection, one may expect the invariant mass to show a spectrum of resonances that have a $\pi\rho$ decay branch. These include the $a_1(1260)$, the $\pi(1300)$, $a_2(1320)$, $a_1(1550)$, etc. The mass spectrum may also show a high mass tail region above these resonances. One measures the reaction rate at normal temperatures for normal mass pion and rho, and intermediate resonances.

Xiong, Shuryak, Brown (XSB) [26] calculate photon production (above 0.7 gev) via the reaction $\pi^- + \rho \rightarrow \pi^- + \gamma$. The reaction $\rho + \pi^- \rightarrow \gamma + \pi^-$ proceeds (according to XSB) through the $a_1(1260)$. XSB estimate Radiative Width ($a_1 \rightarrow \pi\gamma$) = 1.4 MeV, more than two times higher than the experimental value of Zielinski et al. [21]. With this estimated width, they calculate the high energy photon production cross section. They include high temperature effects for a hot hadronic gas. A more recent photon production calculation also involving $a_1$ resonance effects was given by Song [29]. There are many other theoretical studies for gamma rays from hadronic gas and QGP in the Quark Matter conferences, and elsewhere. Some relevant articles are by Ruuskanen [30], Kapusta et al. [31], Alam et al. [32], Nadeau [33], and Schukraft [34]. One can measure such cross sections for normal mass mesons, and therefore to experimentally provide the data base for evaluations of the utility of gamma production experiments in QGP searches. One can experimentally check the $a_1(1260)$ dominance assumption of XSB.

In a hadronic gas at high temperature, the $\pi\rho$ interaction can be near the $a_1$

resonance [26]. One must consider also that certain properties (masses, sizes, parity mixing) of the $\pi$ and $\rho$ and $a_1$ change [26, 29, 35, 36, 37], and that their numbers increase due to the Boltzmann factor. XSB expect an increased yield from the hot hadronic gas, higher than estimated previously by Kapusta et al. [38]. One expects many gamma rays from QGP processes, such as a quark-antiquark annihilation $q\bar{q} \rightarrow g\gamma$ or Compton processes such as $qg \rightarrow q\gamma$ and $\bar{q}g \rightarrow \bar{q}\gamma$. Chakrabarty et al. [39] studied the expected gamma ray yields from hot hadronic gases and the QGP. They suggested that gamma rays between 2-3 GeV from the QGP outshine those from the hot hadronic gas phase.

**D) Investigation of Chiral Anomalies.**

Chiral anomalies can be studied in E781. Before giving experimental details, we first describe chiral anomalies for a massless field theory, following K. Huang [40]. The lagrangian density in such a theory is invariant under a chiral transformation, which implies that a conserved axial-vector current must exist. One may ask if this axial current is the gauge-invariant chiral current. In that case, the divergence of the chiral current should be zero. However, the correct analysis must account for the fact that the currents are singular operators. One can then show that the divergence of the chiral current is equal to an "axial anomaly" term rather than zero. The required conserved axial-vector current does exist and can be defined; but it is not the chiral current, it is not gauge invariant, and it does not couple to physical fields.

For the $\gamma$-$\pi$ interaction at low energy, chiral perturbation theory ($\chi$PT) provides a rigorous way to make predictions; because it stems directly from QCD and relies only on the solid assumptions of spontaneously broken $SU(3)_L \times SU(3)_R$ chiral symmetry, Lorentz invariance and low momentum transfer. Unitarity is achieved by adding pion loop corrections to lowest order, and the resulting infinite divergences are absorbed into physical (renormalized) coupling constants [41, 42]. With a perturbative expansion of the effective Lagrangian limited to terms quartic in the momenta and quark masses ($O(p^4)$), the method successfully describes many physical processes. At $O(p^4)$ level, the lagrangian includes Wess-Zumino-Witten (WZW) terms [43], which incorporate the chiral anomalies of QCD. These modify the Ward identities [41, 44]

for the currents, and also lead to anomalous terms [40, 41, 43, 45] in the divergence equations of the currents. These anomalies at the O($p^4$) level lead directly to an interesting relationship [46] between the processes $\pi^0 \to 2\gamma$ and $\gamma \to 3\pi$. The latter two processes are described by the coupling constants $F_\pi$ and $F_{3\pi}$, respectively. The $F_\pi$ vertex was first described by Adler, Bell, and Jackiw [47]. The relationship is [46]:

$$F_{3\pi} = F_\pi/(ef^2), \tag{3}$$

where e=$\sqrt{(}4\pi\alpha)$ and f is the charged pion decay constant. The experimental confirmation of Eq. 3 would demonstrate that the O($p^4$) terms are sufficient to describe $F_{3\pi}$ within the framework of the chiral anomalies.

For the chiral anomaly, the $\gamma\pi \to \pi\pi$ reaction was measured [45] with 40 GeV pions at Serpukhov via pion pair production by a pion in the nuclear Coulomb field ($\pi^-$ + Z $\to$ $\pi^-$ + Z + $\pi^0$) ; where the incident pion interacts with a virtual photon in the Coulomb field of a nucleus of atomic number Z; and the two final state pions (typically 20 GeV each) were detected in coincidence. This reaction is equivalent to the $\gamma + \pi^- \to \pi^0 + \pi^-$ reaction for a laboratory gamma ray of several hundred MeV incident on a target $\pi^-$ at rest. In the incident pion rest frame, the nucleus Z represents a beam or cloud of virtual photons sweeping past the pion. Such a reaction is an example of the well tested Primakoff formalism [17, 18] that relates processes involving real photon interactions to production cross sections involving the exchange of virtual photons.

In the Serpukhov experiment, it was shown [45] that the Coulomb amplitude clearly dominates and yields sharp peaks in t-distributions at very small four momentum transfers to the target nucleus. The cross sections corresponding to the sharp peaks in the t-distributions for targets with different atomic number Z scaled as $Z^2$, further demonstrating the correspondence with the Primakoff formalism. Background from strong processes (meson or pomeron exchange) has an exponential falloff with increasing t. The Coulomb cross section is about 0.06 $\mu$barns, for a $C^{12}$ target.

To illustrate the kinematics, consider the reaction:

$$\pi + Z \to \pi' + Z' + \pi^{0'} \tag{4}$$

for a 600 GeV incident pion, where Z is the nuclear charge. The 4-momentum of each particle is $P_\pi$, $P_Z$, $P_{\pi'}$, $P_{Z'}$, $P_{\pi^0}$, respectively. In the one photon exchange domain, eqn. 4 is equivalent to:

$$\gamma + \pi \rightarrow \pi' + \pi^0, \tag{5}$$

and the 4-momentum of the incident virtual photon is k = $P_Z$-$P_{Z'}$. The cross section for the reaction of eqn. 4 depends on $F_{3\pi}^2$, and on s, t, $t_1$, $t_0$. Here t is the square of the four-momentum transfer to the nucleus, $\sqrt{s}$ is the invariant mass of the $\pi\pi$ final state, $t_1$ is the square of the 4-momentum transfer between initial and final $\pi^-$ in Eq. 5, and $t_0$ is the minimum value of t to produce a mass $\sqrt{s}$.

The data yield $F_{3\pi} = 12.9 \pm 0.9(stat) \pm 0.5(sys) GeV^{-3}$, from Antipov et al. [45]. The uncertainties do not include aproximately 10% uncertainties [45] in extrapolating $F_{3\pi}$ to threshold (s, $t_1$ approaching zero), which is where Eq. 3 is strictly valid. In addition, Antipov et al. use $f_\pi$ = 90± 5 MeV, and give the theoretical expectation as $F_{3\pi} = 10.5 \pm 1.5 GeV^{-3}$. Comparing experiment and theory, considering the quoted errors, Antipov et al. claimed that the hypothesis of chiral anomalies and color-SU(3) quark symmetry are confirmed. In fact, a recent determination by Holstein [48] of the pion decay constant gave a value of 92.4 ± 0.2 MeV, somewhat lower than the value cited by the Particle Data Group [49] of 93.2 ± 0.1 MeV. Holstein claims that the PDG value 93.2 is too large due to incomplete inclusion of radiative corrections in its extraction. The Holstein value was confirmed independently by Marciano and Sirlin [50]. Both the Holstein and PDG values and errors are significantly different from the value 90 ± 5 MeV used by Antipov et al. In what follows, we use the value f= 92.4 ± 0.2 MeV rather than the 1990 and 1992 PDG value; as this appears to be very well founded [48, 50], and also leads to more conservative conclusions. The consequently revised O($p^4$) expectation for $F_{3\pi}$ is therefore 7.4% lower than given by Antipov et al., and also the uncertainty associated with f is reduced, leading to $F_{3\pi} = 9.72 \pm 0.06 GeV^{-3}$. In that case, the experimental result of Antipov et al. in fact differs with the O($p^4$) chiral anomaly expectation by at least two standard deviations. A related reaction [51] to determine $F_{3\pi}$ is $\pi^- + e \rightarrow \pi^- + \pi^0 + e'$, for which an incident high energy pion scatters inelastically from a target electron in an

atomic orbit. The data uncertainties [51] in this case are roughly 25%, and there are also additional theoretical uncertainties in the extrapolation to zero momentum transfer. Therefore, the hypothesis of chiral anomalies at $O(p^4)$ is not confirmed by the available $\gamma \to 3\pi$ data.

Bijnens et al. [52, 53, 54] within $\chi$PT studied yet higher order corrections in the abnormal intrinsic parity (anomalous) sector. They included one-loop diagrams involving one vertex from the WZW term, and tree diagrams from the $O(p^6)$ lagrangian. Some one-loop diagrams give finite contributions. Others lead to divergences that are eliminated by the $O(p^6)$ terms. These higher order corrections are small for $F_\pi$. For the $F_{3\pi}$ vertex, they increase the lowest order value of $F_{3\pi}$, from Eq. 3, from 7% to 12%. Bijnens et al. give the eq. 3 value of $F_{3\pi}$ as 9.5 GeV$^{-3}$, which corresponds to using the PDG value f=93.2 MeV. The one-loop and $O(p^6)$ corrections to $F_{3\pi}$ are comparable in strength. The loop corrections to $F_{3\pi}$ are not constant over the whole phase space, due to dependences on the momenta of the 3 pions. The average effect is roughly 10%, which then changes the theoretical prediction from Eq. 3 to roughly 11. GeV$^{-3}$. As discussed by Bijnens et al., the higher order corrections improve the agreement between theoretical predictions and the data. The large experimental errors however do not allow one to disentangle the loop effects from the $O(p^6)$ effects. The calculations of Bijnens et al motivate an improved experiment.

The experiment at 40 GeV suffered from the need to disentangle Primakoff and strong contributions. This problem was a major factor in setting the systematic uncertainty of the experiment. In E781, at the 600 GeV higher energy, the strong contribution is negligible, which should significantly reduce the systematic uncertainty. The 1 MHz pion flux at FNAL E781 will enable superb statistics for a new measurement. Also, the extrapolation to threshold can be accomplished with significantly smaller error in future experiments. This is done for a high statistics experiment by restricting the data set to significantly lower values of $\sqrt{(s)}$ and $t_1$, compared to Antipov et al. One can therefore get improved data for a significantly improved test of chiral anomalies, with E781. One can test how well $\chi$PT works in the anomalous sector. How anomalous is the real world anyhow?

**CONCLUSIONS**

This completes the discussion of points A-D. There are experimental possibilities at FNAL E781, and elsewhere. The cross sections need to be studied experimentally versus A, incident energy, incident particle type, $\Delta p_T$, $\Sigma p_T$, invariant mass and transverse mass of the produced two-meson or multi-pion final systems. Theoretical calculations are available for many of the reactions discussed, and interesting E781 data should motivate further theoretical developments.

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# References

[1] J. Russ, spokesman, FNAL E781 SELEX proposal, Collaborating Institutions: Carnegie-Mellon, Fermilab, Beijing, SPNPI (Gatchina), ITEP (Moscow), IHEP (Protvino), U. Iowa, Sao Paulo, Rio de Janerio, U. Bristol, U. Washington, U. Rochester, MPI Heidelberg, and Tel Aviv U. The E781 experiment involves charm baryon production and decay, and also Primakoff physics.

[2] M.Zielinski et al., Z.Phys.C, **16** ,197 (1983).

[3] B. Collick et al., Phys. Rev. Lett. 53 (1984) 2374.

[4] L. Frankfurt, G. A. Miller, and M. Strikman, Phys.Rev. Lett. 71, 2859 (1993).

[5] B. Blattel et al., Phys. Rev. 47D (1993) 2761.

[6] B. Blattel et al., Phys. Rev. Lett. 70, 896 (1993).

[7] L. Frankfurt and M. Strikman, Phys. Rev. Lett. 66, 2289 ( 1991).

[8] L. Frankfurt, G. A. Miller, M. Strikman, U. Washington and Penn. State U. preprint DOE/ER/40427-06-N94, Color Optics Review, submitted for publication March 1994, Ann. Rev. Nucl. Part. Sci.

[9] G. Baym, L. Frankfurt and M. Strikman, Nucl. Phys. A566 (1994) 149C.

[10] E. L. Feinberg and I. Y. Pomeranchuk, Suppl. Nuovo Cimento III, (1956) 652.

[11] M. L. Good and W. D. Walker, Phys. Rev. 120 (1960) 1857.

[12] L. Frankfurt, G. A. Miller, M. Strikman, Phys. Lett. B304 (1993) 1.

[13] A. B. Zamolodchikov, B. Z. Kopeliovich, L. I. Lapidus, JETP Lett. 33 (1981) 595; B. Z. Kopeliovich, Sov. J. Part. Nucl. 21 (1990) 49.

[14] G. Bertsch et al., Phys. Rev. lett. 47 (1981) 297.

[15] W.Mollet et al., Phys.Rev.Lett. **39**, 1646 (1977).

[16] B. Kopeliovich, private communication, 1992.

[17] T. Jensen *et al.*, Phys. Rev. 27D, 26 (1983).

[18] M. Zielinski *et al.*, Phys. Rev. 29D, 2633 (1984).

[19] J. Huston et al.,Phys. Rev. 33 (1986) 3199.

[20] L. Capraro et al., Nucl. Phys. B288 (1987), 659.

[21] M. Zielinski et al., Phys. Rev. Lett. 52 (1984) 1195.

[22] S. Cihangir et al., Phys. Lett. 117B (1982) 119.

[23] M.A. Moinester, Proceedings of the Conference on the Intersections Between Particle and Nuclear Physics, Tucson, Arizona, 1991, AIP Conference Proceedings 243, P. 553-558, 1992, Ed. W. Van Oers.

[24] H. J. Lipkin, M. A. Moinester, Phys. Lett. 287B (1992) 179-184.

[25] B. R. Holstein, Comments Nucl. Part. Phys. 19 (1990) 239.

[26] L. Xiong, E. Shuryak, G. Brown, Phys. Rev. 46D (1992) 3798.

[27] L. Landsberg, Sov. J. Nucl. Phys. 55 (7), (1992), 1051.

[28] M. Zielinski et al., Z. Phys. C-Particles and Fields 31,545 (1986).

[29] C. Song, Phys. Rev. 47C (1993) 2861.

[30] P. V. Ruuskanen, QM91, Nucl. Phys. 544C (1992) 169C.

[31] J. Kapusta et al., QM91, Nucl. Phys. 544C (1992) 485C;
J. Kapusta, QM93, Nucl. Phys. 566A (1994) 45C.

[32] J. Alam et al., QM91, Nucl. Phys. 544C (1992) 493C.

[33] H. Nadeau, Phys. Rev. 48D (1993) 3182.

[34] J. Schukraft, QGP Review Article, J. Phys. G, Nucl. Part. Phys. 19 (1993) 1705.

[35] M. Dey et al.,Phys. Lett. 252B (1990) 620.

[36] M. Asakawa and M.C. Ko, Nucl. Phys. 560A (1993) 399.

[37] M. Herrmann, B. L. Friman, W. Norenberg, Nucl. Phys. 560A (1993) 411.

[38] J. Kapusta et al.,Phys. Rev. 44D (1991) 2774; H. Nadeau et al., Phys. Rev. 45C (1992) 3034.

[39] S. Chakrabarty et al., Phys. Rev. 46D (1992)3802; Nucl. Phys. A544 (1992) 493.

[40] K. Huang, in "Quarks, Leptons, and Gauge Fields", 2nd edition, World Scientific, 1992, Capter 11, "The Axial Anomaly".

[41] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158 (1984)142; Nucl. Phys. 250B (1985) 465; H. Leutwyler, "On the foundations of chiral perturbation theory", University of Bern Institute of Theoretical Physics preprint BUTP-93/24, Aug. 1993, Bulletin Board: HEP-PH@xxx.LANL.GOV - 9311274

[42] J. F. Donoghue and B. R. Holstein, Phys. Rev. 40D (1989) 2378.

[43] J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95;
E. Witten, Nucl. Phys. B223 (1983) 422;
W. A. Bardeen, Phys. Rev. 184 (1969) 1848;
J. F. Donoghue and D. Wyler, Nucl. Phys. 316B (1989) 289;
R. Akhoury and A. Alfakih, Ann. Phys. 210 (1991) 81;
C. Kuang-Chao et al., Phys. Lett. 134B (1984) 67;
J. L. Manes, Nucl. Phys. B250 (1985) 369.

[44] B. Zumino et al., Nucl. Phys. B239 (1984) 477.

[45] Y. M. Antipov et al., Phys. Rev. 36D (1987) 21.

[46] M. V. Terentev, Sov. J. Nucl. Phys. 15 (1972) 665;
P. G. O. Freund and A. Zee, Phys. Lett. 132B (1983) 419;
G. Maiella, Phys. Lett. 155B (1985) 121;
S. Rudaz, Phys. Rev. 10D (1974) 3857; Phys. Lett. 145B (1984) 281.

[47] S. Adler, Phys. Rev. 177 (1969) 2426;
J. Bell, R. Jackiw, Nuovo Cimento 60A (1969) 47.

[48] B. R. Holstein, Phys. Lett. 244B (1990) 83.

[49] Particle data Group, J. J. Hernandez et al., Phys. Lett. B239 (1990) 1; K. Hikasa et al., Phys. Rev. 45D (1992) 1 (P. V11.1)

[50] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. 71 (1993) 3629.

[51] S. R. Amendolia et al., Phys. Lett. 155B (1985) 457.

[52] J. Bijnens, A. Bramon, F. Cornet, Phys. Lett. 237B (1990) 488.

[53] J. Bijnens, A. Bramon, F. Cornet, Z. Phys. C46 (1990) 599.

[54] J. Bijnens, Chiral Perturbation Theory and Anomalous Processes, (Review), Int. Journal Mod. Phys. A, Vol. 8 (1993) 3045.